Analysis of Multi-Phase Permanent-Magnet Synchronous Machines

T.J.E. Miller and M.I. McGilp
SPEED Laboratory, University of Glasgow, UK

Abstract — Large AC machines are sometimes fed by multiple inverters. This paper presents the complete steady-state analysis of the PM synchronous machine with multiplex windings, suitable for driving by multiple independent inverters. Machines with 4, 6 and 9 phases are covered in detail. Particular attention is given to the magnetic interactions not only between individual phases, but between channels or groups of phases. This is of interest not only for determining performance and designing control systems, but also for analysing fault tolerance. It is shown how to calculate the necessary self- and mutual inductances and how to reduce them to a compact dq-axis model without loss of detail.

Index Terms — multi-phase machines, permanent-magnet machines.

I. INTRODUCTION

A. Multiple phases

The advantages of polyphase machines and systems are well known and long established. For example the 3-phase 3-wire system conveys 50% more power with only 87.9% of the conductor area, compared with a 2-phase 3-wire system, [1]. In electrical machines the winding factors can be increased, and the torque ripple reduced, by using multiple phases. Space-harmonics in the airgap flux are also reduced.

B. Multiplex windings

A multiplex winding of plex x has x m-phase sets of balanced polyphase windings, giving mx phases. For example Fig. 1(a) shows a 6-phase winding that is a duplex 3-phase winding, with m = 3 and x = 2. Fig. 1(b) shows a 4-phase winding with m = 2 and x = 2. A tripplex winding with m = 3 would have 9 phases. Most conventional machines have simplex windings with x = 1, including the 5-phase machines studied by Parsa and Toliyat [4,5]. Even for induction motors there appears to be very little published work on multiplex machines [6,7]; and evidently none on PM machines.

In practice the displacement \( \alpha \) between the sets must be chosen so that the fluxes in common sections of the magnetic circuit make sensible utilization of the available cross-section, and to make the slot-fill factors even. We shall see later that the mutual coupling between sets imposes further constraints on \( \alpha \).

C. Reasons for using multiplex windings

The increased phase number extends the advantages of the simplex machine mentioned earlier, but additional advantages can arise with very large machines.

II. BASIC ANALYSIS

The windings of the 6-phase machine in Fig. 1 are labelled \( a,b,c \), and \( a_1,b_1,c_1 \). Each 3-phase winding is wye-connected and is balanced in itself. In general all six phases are mutually coupled. If the rotor rotates anticlockwise, the voltages and currents in set 2 lead those in set 1 by the phase angle \( \alpha \), which is the angular displacement between the two 3-phase windings.
The rotor $d$-axis is at the angle $\Theta_r$ relative to the axis of phase $a_1$, and at $\Theta_2$ relative to the axis of phase $a_3$, and

$$\alpha = \Theta_2 - \Theta_1.$$  

(1)

The $q$-axis leads the $d$-axis by $\pi/2$ electrical radians. The $dq$ transformation applied to $a_1b_1c_1$ gives

$$v_{d1} = R_1i_{d1} + p\psi_{d1} - \omega\psi_{q1};$$

$$v_{q1} = R_1i_{q1} + p\psi_{q1} + \omega\psi_{d1}. $$

(2)

Normally with only 3 phases, there is just one $d$-coil and one $q$-coil and the flux-linkages are given by

$$\psi_{d1} = \Psi_{M1d} + L_{d1}i_{d1};$$

$$\psi_{q1} = L_{q1}i_{q1}, \quad \Psi_{M1d}$$

where $\Psi_{M1d}$ is the flux-linkage produced by the magnet. We are going to transform the second winding $a_2b_2c_2$ into a second pair of coils $d_2q_2$, on the same direct and quadrature axes as the $d_1q_1$ coils, and these will be coupled to the $d_1q_1$ coils, so we can write

$$\psi_{d2} = \Psi_{M2d} + L_{d2}i_{d2} + M_{d1d2}i_{d1};$$

$$\psi_{q2} = L_{q2}i_{q2} + M_{q1q2}i_{q1}. $$

(4)

The model can be completed by adding the equations for the $a_2b_2c_2$ winding corresponding to (2) and (4):

$$v_{d2} = R_2i_{d2} + p\psi_{d2} - \omega\psi_{q2};$$

$$v_{q2} = R_2i_{q2} + p\psi_{q2} + \omega\psi_{d2}.$$  

(5)

and

$$\psi_{d2} = \Psi_{M2d} + L_{d2}i_{d2} + M_{d1d2}i_{d1};$$

$$\psi_{q2} = L_{q2}i_{q2} + M_{q1q2}i_{q1}. $$

(6)

The mutual inductance $M_{d1d2}$ or $M_{d2d1}$ between the $d_1$ and $d_2$ coils is important, and we must consider whether it depends on the displacement angle $\alpha$. Later we will also consider the possibility of cross-coupling between the $d$ and $q$-axes, which is absent from (4) and (6).

If $\alpha = 0$, and if all phases have the same effective number of turns, then phases $a_1$ and $a_2$ will be aligned with the $d$-axis at the same time. In this case we expect $M_{d1d2}$ to be equal to $L_{mrd}$, the magnetizing component of the synchronous inductance $L_d$. Plus another term $m_{o12d}$ arising from mutual coupling in the slots and in the end-windings:

$$M_{d1d2} = L_{mrd} + m_{o12d}(\alpha).$$

(7)

The mutual inductance term $m_{o12d}$ may include a component due to differential or harmonic leakage, while $L_{mrd}$ is related to the conventional self- and mutual leakage inductances $L_r$ and $M_{rd}$, and is also associated with the fundamental ampere-conductor distribution.

Any relationship between $L_r$, $M_o$, and $m_{o12d}$ is not immediately obvious, but $m_{o12d}$ can be calculated formally by applying the $dq$-axis transformation to the entire inductance matrix as shown below.

If $\alpha = \pi/2$, phase $a_2$ will be aligned with a $q$-axis of the rotor when phase $a_1$ is aligned with a $d$-axis. If $L_d$ and its components $L_{mrd}$, $L_o$, and $M_r$ are independent of the orientation of the $d$-axis relative to the winding axes, (7) remains valid, although the value of $m_{o12d}$ will be different.

By similar reasoning for the $q$-axis coils,

$$M_{q1q2} = L_{mq} + m_{o12q}(\alpha).$$

(8)

In general, the currents (and voltages) in the second set must be phase-shifted by $\alpha$ relative to those in the first set.

In a duplex wye-connected 6-phase machine, there are only 2 coils on the $d$-axis and 2 coils on the $q$-axis, and the two sets of phases will be balanced as long as $M_{d1d2} = M_{d2d1}$ and $M_{q1q2} = M_{q2q1}$, the normal condition of reciprocity.

However, consider the extension to a 9-phase machine with 3 coils on the $d$-axis and 3 on the $q$-axis. We will have

$$M_{d1d2} = L_{md} + m_{o12d}(\alpha);$$

$$M_{q1q2} = L_{mq} + m_{o12q}(\alpha).$$

(9)

If $m_{o12d}$, $m_{o23d}$ and $m_{o13d}$ are not all equal, it will be impossible to achieve balance between the three sets of windings. Even if they are all supplied with the same currents phase-shifted by $0$, $\alpha$, and $2\alpha$ respectively, the voltages will be unequal because of the differences in the mutual inductances. In general, we can say that a winding of higher multiplicity ($x > 2$), cannot be balanced unless all the $d$-axis mutual inductances are equal; and all the $q$-axis mutual inductances are separately equal. To pursue this further, we need a matrix analysis of the inductances.

III. MATRIX ANALYSIS

In terms of flux-linkages $[\Psi]$ and currents $[i]$, the $dq$-axis transformation for plain 3-phase machines is summarized as follows, where $[T]$ is the transform matrix used in [1-3]:

$$[\psi_{abc}] = [L_{abc}][i_{abc}];$$

(10)

$$[i_{dq0}] = [T][i_{abc}]; \quad [i_{abc}] = [T]^{-1}[i_{dq0}],$$

(11)

$$[\psi_{dq0}] = [T][\psi_{abc}] = [T][L_{abc}][T]^{-1}[i_{dq0}]$$

$$= [L_{dq0}][i_{dq0}].$$

(12)

The matrix $[L_{abc}]$ is well known to contain constant terms and second-harmonic terms in rotor position $\theta$, and the familiar result is

$$[L_{dq0}] = \begin{bmatrix}
L_d & L_q \\
L_q & -L_o
\end{bmatrix}$$

where $L_d$ and $L_q$ are the synchronous inductances.

To consider the duplex 6-phase machine we use a full square inductance matrix (15) of the following form, of order $6 \times 6$, partitioned between the two sets of phases:

$${\begin{bmatrix}
  a_1b_1c_1 & a_1b_2c_2 \\
  [A_1] & [B] \\
  a_2b_1c_2 & [A_2]
\end{bmatrix}}$$

(14)
The full form of this matrix is

\[
\begin{bmatrix}
    a_1 & b_1 & c_1 \\
    a_2 & b_2 & c_2 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
    L_{a1} & M_{a1b} & M_{a1c} \\
    M_{b1a} & L_{b1} & M_{b1c} \\
    M_{c1a} & M_{c1b} & L_{c1} \\
\end{bmatrix}
\begin{bmatrix}
    a_1 \\
    a_2 \\
\end{bmatrix} = \begin{bmatrix}
    M_{a1b} & M_{a1c} & M_{a12} \\
    M_{b1a} & L_{b1} & M_{b12} \\
    M_{c1a} & M_{c1b} & L_{c1} \\
\end{bmatrix}
\begin{bmatrix}
    b_1 \\
    b_2 \\
\end{bmatrix} + \begin{bmatrix}
    M_{a1c} \\
    M_{b1c} \\
    M_{c1c} \\
\end{bmatrix}
\begin{bmatrix}
    c_1 \\
    c_2 \\
\end{bmatrix}
\]

If \( [B] = 0 \), there is no mutual coupling between the \( a, b, c \) windings and the \( a, b, c \) windings. In that case the dq0 transformation \( [T_j] = [T(0)] \) can be applied to \( [A_i] \), and \( [T_j] = [T(0)] \) to \( [A_i] \), and the result is two completely independent, uncoupled dq0 systems.

\[
\begin{bmatrix}
    d_1q_0, \delta_1 \\
    d_2q_0, \delta_2 \\
\end{bmatrix} = \begin{bmatrix}
    [T_1][A_1][T_1]^{-1} \\
    [T_2][A_2][T_2]^{-1} \\
\end{bmatrix}
\]

In practice the \( dq0 \) system is mutually coupled to the \( dq0 \) system, and we have already postulated the form of the mutual inductances in (7), (8) and (9). The situation so far can be summarized by writing the duplex dq0 inductance matrix as follows.

\[
\begin{array}{c|cc|c|cc}
   & a_1 & b_1 & c_1 & a_2 & b_2 & c_2 \\
\hline
   a_1 & L_{a1} & M_{a1b} & M_{a1c} & M_{a12} \\
   b_1 & M_{b1a} & L_{b1} & M_{b1c} & M_{b12} \\
   c_1 & M_{c1a} & M_{c1b} & L_{c1} & M_{c12} \\
\hline
   a_2 & M_{a12} & M_{a1b} & M_{a1c} & M_{a12} \\
   b_2 & M_{b12} & M_{b1b} & M_{b1c} & M_{b12} \\
   c_2 & M_{c12} & M_{c1b} & M_{c1c} & M_{c12} \\
\hline
\end{array}
\]

The notation \([T(\theta)]\) means that the elements of \([T]\) are trigonometric functions of rotor position \( \theta \), as is known; see [1]-[3].

To the complete 6-way matrix \( a_1b_1c_1a_2b_2c_2 \), we get

\[
\begin{bmatrix}
    d_1q_0, \delta_1 \\
    d_2q_0, \delta_2 \\
\end{bmatrix} = \begin{bmatrix}
    [T_1][A_1][T_1]^{-1} \\
    [T_2][A_2][T_2]^{-1} \\
\end{bmatrix}
\]

\[ (16) \]

\[
\begin{array}{c|c|c|c|c|c|c}
   & a_1 & b_1 & c_1 & a_2 & b_2 & c_2 \\
\hline
   a_1 & \lambda & \mu & \nu \\
   b_1 & \nu & \lambda & \mu \\
   c_1 & \mu & \nu & \lambda \\
\hline
   a_2 & \lambda & \nu & \mu \\
   b_2 & \mu & \lambda & \nu \\
   c_2 & \nu & \mu & \lambda \\
\hline
\end{array}
\]

Thus for the upper-right partition,

\[
[T_1][B][T_2]^{-1} = [T_1] \cdot \begin{bmatrix}
    \lambda & \mu & \nu \\
    \nu & \lambda & \mu \\
    \mu & \nu & \lambda \\
\end{bmatrix} \cdot [T_2]^{-1}
\]

\[ (21) \]

Without writing out the entire solution, it is instructive to work out and examine just the \((1,1)\)th element, which represents \( m_{o12d} \):

\[
[T_1][B][T_2]^{-1}(1,1) = m_{o12d} = \lambda \cos (\theta_2 - \theta_1) + \\
+ \mu \cos (\theta_2 - \theta_1 - 120^\circ) \\
+ \nu \cos (\theta_2 - \theta_1 + 120^\circ).
\]

\[ (22) \]

Since \( \theta_2 - \theta_1 = \alpha \), we can write this as

\[
m_{o12d} = \lambda \cos \alpha + \\
+ \mu \cos (\alpha - 120^\circ) \\
+ \nu \cos (\alpha + 120^\circ).
\]

\[ (23) \]

By the same process we find that

\[
[T_1][B][T_2]^{-1}(2,2) = m_{o12d} = \\
= \lambda \cos \alpha + \mu \cos (\alpha - 120^\circ) + \nu \cos (\alpha + 120^\circ),
\]

\[ (24) \]
This shows the feature we might have hoped to avoid: that although \( m_{q12d} = m_{q12a} \), these mutual inductances appear to depend on \( \alpha \), and further analysis is necessary to find out the constraints that apply to the choice of \( \alpha \) for duplex, triplex, and higher multiplex windings.

For the duplex winding we have already observed that it does not matter if \( m_{q12d} \) and \( m_{q23d} \) depend on \( \alpha \), because there is only one mutual inductance on the \( d \)-axis and one on the \( q \)-axis.

For triplex windings we can expect \( m_{q12d} \) and \( m_{q23d} \) both to be given by (23) directly, because the displacement angle between sets 1 and 2 is the same as the displacement angle between sets 2 and 3, both being equal to \( \alpha \). But the displacement angle between sets 1 and 3 is \( 2\alpha \), so we expect \( m_{q31d} \) to be given by (23) with \( 2\alpha \) substituted in place of \( \alpha \). Now we know that it is necessary to have

\[
m_{q12d} = m_{q23d} = m_{q31d}
\]

(25)
to operate a triplex winding balanced. So what we really need, at this stage, is to understand the conditions under which (23) gives the same value for \( 2\alpha \) as it does for \( \alpha \).

Now consider the mutual inductances \( \lambda, \mu, \nu \) in terms of the winding harmonics. From Fig. 3 we can see that \( \lambda \) is a function of the angle \( \alpha \), that is, \( \lambda = \lambda(\alpha) \). Similarly \( \mu \) is a function of the angle \( \alpha - 120^\circ \), and if we assume the same functional dependence, we can write \( \mu = \mu(\alpha - 120^\circ) \). Finally \( \nu = \nu(\alpha + 120^\circ) \), again with the same functional dependence on the angle. For each harmonic component of the inductance, let

\[
\begin{align*}
\lambda &= \Lambda_\alpha \cos n\alpha; \\
\mu &= \Lambda_\alpha \cos (n(\alpha - 120^\circ)); \\
\nu &= \Lambda_\alpha \cos (n(\alpha + 120^\circ)).
\end{align*}
\]

This is, in effect, one term of a Fourier series expansion of the inductances \( \lambda, \mu, \nu \). If we substitute this in (23) we get the total \( n^\text{th} \) harmonic inductance

\[
m_n(\alpha) = \Lambda_\alpha \cos n\alpha \cos \alpha \\
+ \cos n(\alpha - 120^\circ) \cos (\alpha - 120^\circ) \\
+ \cos n(\alpha + 120^\circ) \cos (\alpha + 120^\circ).
\]

(27)

It can be shown that \( m_n(2\alpha) = m_n(\alpha) \) for \( \alpha = 20^\circ, 40^\circ, 60^\circ, 80^\circ, 100^\circ, 120^\circ \) etc., but not for \( \alpha = 15^\circ \) or \( 30^\circ \).

This implies that \( \alpha = 20^\circ, 40^\circ \) or even \( 80^\circ \) is acceptable for triplex (9-phase) windings, but not \( 15^\circ \) or \( 30^\circ \). On the other hand \( 15^\circ \) and \( 30^\circ \) are acceptable for duplex (6-phase) windings. In general it appears that \( \alpha = 180k/60\pi \) gives the required result expressed by eqn. (25), at least for duplex and triplex cases.

It is of interest to give some thought to the conditions that would make \( \lambda = \mu = \nu = 0 \). If this could be achieved, then \textit{any} value of the displacement angle \( \alpha \) would be acceptable with any value of \( x \).

If all the mutual coupling between phases of different sets were via the slot-leakage flux, this condition could be satisfied by ensuring that the windings of \( a_i, b_i, c_i \) have no shared slots with \( a_j, b_j, c_j \). This is more troublesome, since it is a function of several space-harmonics of MMF having different pole-pitches. For example, if \( \alpha = 30^\circ \) there should be no third-harmonic linkage between \( a_1 \) and \( a_2 \), since \( 3 \times 30 = 90^\circ \), rendering \( a_1 \) and \( a_2 \) orthogonal and therefore uncoupled for this harmonic.

The same cannot be said of the non-triplen harmonics \( 5^\circ, 7^\circ \) etc. One might hope that the differential leakage is small; but the very nature of multiplex windings is to reduce the number of slots per pole per phase, potentially increasing the winding factors of some of the harmonics we would like to eliminate. The conclusion is that it is safest to select \( \alpha \) from the preferred values indicated on the previous page, when designing triplex or higher-multiplex windings.

Finally let us test the symmetry of the combined dq0 matrix by evaluating the \( [1,1] \)th term of \( [T_1][B][T_1]^\text{T} \), the lower-left partition. The evaluation is similar to that of the upper-right partition \( [T_1][B][T_1]^\text{T} \), but with \( \theta_1 \) and \( \theta_2 \) interchanged. The result is

\[
m_{q12d} = \lambda \cos \alpha + \\
\mu \cos (\alpha - 120^\circ) + \nu \cos (\alpha + 120^\circ).
\]

(28)

Thus

\[
m_{q12d} = m_{q12d}
\]

(29)

and the mutual (off-diagonal) inductances in the dq0 matrix are indeed reciprocal, as would be expected.

In order to proceed with manageable calculations, the simplest expedient is to drop \( m_{q12d} \) from eqn. (7) and \( m_{a12q} \) from its \( q \)-axis counterpart, leaving

\[
M_{d1d} = L_{md}
\]

(30)

and

\[
M_{q1q2} = L_{mq}.
\]

(31)

Alternatively, \( m_{q12d} \) and \( m_{a12q} \) can be retained in the equations but treated as a user-defined “perturbation” parameter. This permits an experimental numerical approach to determine how large these parasitic mutual inductances must be to cause significant errors in the calculation (and/or to cause significant imbalance between winding sets in cases of multiplicity 3 or higher). This approach is followed in the remaining sections.
IV. TORQUE

The electromagnetic torque is the result of interaction between the currents and the rotational voltages in (2) and (5). For the first set of windings,

$$T_1 = m p \left[ \Psi_d I_{d1} + \Psi_q I_{q1} \right],$$

where \( m \) is the number of phases (3) in the set, and \( p \) is the number of pole-pairs. Substituting for the flux-linkages from (3),

$$T_1 = m p \left[ \Psi_m I_{d1} + (L_{d1} - L_{q1}) I_{d1} I_{q1} + M_{d1q1} I_{d1} I_{q1} - M_{q1d1} I_{q1} I_{d1} \right].$$

For the second set of windings the process is the same:

$$T_2 = m p \left[ \Psi_m I_{q2} + (L_{d2} - L_{q2}) I_{d2} I_{q2} + M_{d2q2} I_{d2} I_{q2} - M_{q2d2} I_{q2} I_{d2} \right].$$

In each of (33) and (35), the first term is the familiar permanent-magnet alignment torque, while the second term is the familiar reluctance torque. The third and fourth terms arise from interaction between the two sets of windings: \( d \)-axis flux produced by one set interacts with the \( q \)-axis current of the other set. Consequently four additional terms appear for a duplex winding; eight for a triplex winding, and \( 4(x - 1) \) for an \( x \)-plex winding.

The torque equations (33) and (35) can be used for transient or steady-state torque. In the steady-state they can be written in terms of phasors (RMS AC quantities):

$$T_1 = \frac{m p}{\omega} \left[ E_{d1} I_{d1} + (X_{d1} - X_{q1}) I_{d1} I_{q1} + X_{d1q1} I_{d1} I_{q1} - X_{q1d1} I_{q1} I_{d1} \right];$$

$$T_2 = \frac{m p}{\omega} \left[ E_{q2} I_{q2} + (X_{d2} - X_{q2}) I_{d2} I_{q2} + X_{d2q2} I_{d2} I_{q2} - X_{q2d2} I_{q2} I_{d2} \right].$$

where \( \omega = 2\pi f \) and \( X = \omega L \). Regrouping the terms of this equation, we can separate the alignment torques and the reluctance torques as

$$T_{ei} = \frac{m p}{\omega} \left[ E_{q1} I_{q1} + E_{d1} I_{d1} \right];$$

$$T_{eA} = \frac{m p}{\omega} \left[ (X_{d1} - X_{q1}) I_{d1} I_{q1} + (X_{d2} - X_{q2}) I_{d2} I_{q2} \right];$$

$$T_{eB} = \frac{m p}{\omega} \left[ (X_{d1q1} - X_{q1d1}) I_{d1} I_{q1} + (X_{d2q2} - X_{q2d2}) I_{d2} I_{q2} \right];$$

where \( T_{ei} \) is the total alignment torque, \( T_{eA} \) is the sum of the “self” reluctance torques of the two sets operating individually, and \( T_{eB} \) is the sum of the “mutual” reluctance torques that arise from mutual coupling between the two sets of phases.

With a triplex winding the result is given by (36):

$$T_{ei} = \frac{m p}{\omega} \left[ E_{d1} I_{d1} + E_{d2} I_{d2} + E_{d3} I_{d3} \right];$$

$$T_{eA} = \frac{m p}{\omega} \left[ (X_{d1} - X_{q1}) I_{d1} I_{q1} + (X_{d2} - X_{q2}) I_{d2} I_{q2} + (X_{d3} - X_{q3}) I_{d3} I_{q3} \right];$$

$$T_{eB} = \frac{m p}{\omega} \left[ (X_{d1q1} - X_{q1d1}) I_{d1} I_{q1} + (X_{d2q2} - X_{q2d2}) I_{d2} I_{q2} + (X_{d3q3} - X_{q3d3}) I_{d3} I_{q3} \right].$$

V. STEADY-STATE PHASOR DIAGRAM

Under AC steady-state conditions the RMS values of the \( d \)- and \( q \)-axis flux-linkages \( \Psi_d \) and \( \Psi_q \) in (3) can be combined into a phasor

$$\Psi_1 = \Psi_{d1} + j \Psi_{q1},$$

and likewise the current can be expressed as a phasor

$$I_1 = I_{d1} + j I_{q1}.$$

The voltage phasor is then given by

$$V_1 = V_{d1} + j V_{q1} = R_1 I_1 + j \omega \Psi_1,$$

in which

$$V_{d1} = R_1 I_{d1} + X_{q1} I_{q1} - X_{q1d1} I_{d1} - X_{d1q1} I_{q1};$$

$$V_{q1} = E_{q1} + R_1 I_{q1} + X_{d1} I_{d1} + X_{d1d1} I_{d1}.$$
The cross-coupling terms appear in the phasor diagram as additional voltage-drops which tend to limit the current. If \( \alpha = 0 \), we have tightly coupled inductances between the two sets, as already observed; and if these sets are fed from a common voltage source the current in each set will be approximately half the current that would flow in one set if the other were open-circuited. This is an important practical point because it implies that in a duplex winding, if one set is open-circuited, the current in the other set could increase by a factor approaching 200%, if it were not regulated. Likewise if one set is short-circuited, the impedance of the second set will be reduced and its current could also increase to a high value if it is not regulated.

The behaviour of the duplex sets is analogous to that of parallel inductances, Fig. 5, with an equivalent inductance

\[
\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} \tag{44}
\]

If \( L_1 = L_2 = L \), this simplifies to \((L + M)/2\), and the equivalent circuit is two parallel uncoupled inductances, each of value \(L + M\).

When \( \alpha = 0 \), \( M \) becomes close to \( L \) and the equivalent inductance is approximately \((L + M)/2 = L\). The total current is that which is limited by \(L\), and half the current flows in each set. But if one set is open-circuited, the same total current will tend to flow in one set. The implication is that regulation of the current is essential.

We are now in a position to solve the system with any given voltages or currents applied to the terminals of the two sets of windings. In the steady state this is a question of solving eqns. (42) and (43) when either the voltages \(V_1\) and \(V_2\) are given, or the currents \(I_1\) and \(I_2\) are given.

The authors thank Dr. Mircea Popescu of Motor Design Ltd. (formerly of the SPEED Laboratory), and H. C. Karmaker of TECO Westinghouse and for their assistance.

**REFERENCES**


**VI. CONCLUSION**

The main results of this paper are the phasor diagram of the multiplex permanent-magnet synchronous machine, and the torque equation. Both these tools are highly useable in design work in much the same way as the simpler versions that are familiar with conventional simplex machines, and both of them provide clear physical interpretation of the interaction between the sets or channels. The \(dq\)-axis model has additional mutual reactances between the sets, and these can be calculated in the first instance from the full inductance matrix transformed into \(dq\) axes.

It is shown that the sets of a symmetrical duplex winding are inherently balanced; but with triplex windings and windings of higher multiplicity the displacement angle between sets is critical, and an expression is given for it.

Although the multiplex or \(x\)-plex machine appears suited to the use of multiple inverters, each of 1/\(\alpha\) th the full rating, it is not inherently fault-tolerant because the sets are always magnetically coupled. Therefore, any operation with unbalanced loads between sets must be controlled. Further work on this topic would be useful.

The analysis does lead to a practical set of equations for solution, in spite of the lengthy mathematics of their derivation. Future work might well include a more detailed analysis of the mutual reactances between sets (preferably not by finite-element grinding).

**ACKNOWLEDGMENT**

The authors thank Dr. Mircea Popescu of Motor Design Ltd. (formerly of the SPEED Laboratory), and H. C. Karmaker of TECO Westinghouse and for their assistance.